Resolution of the $R_b - \alpha_s$ crisis of the SM in an exotic ETC scenario.

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Abstract

In this article we show that the $R_b - \alpha_s$ crisis of the SM can be resolved in a commuting TC scenario with sideways gauge bosons only if the ETC representation contains mirror technifermions. For a toy ETC model we estimate the value of $\xi_t^2 = g_{E,L}/g^U_{E,R}$ that is needed to produce the observed LEP excess in R_b. It is also shown that the same value of ξ_t^2 resolvs the α_s crisis of the SM but produces a $\delta\rho_{new}$ which is barely within the present experimental bound.

The SM works extremely well. All experimental data are quite well explained in the context of the SM. However the precision electroweak measurements at LEP have recently produced data about three observables $R_b, R_c and \alpha_s(M_z)$ that seem to show some deviations from the SM predictions which might well be the first hint of new physics beyond the SM. The first and most widely reported deviation is the observed LEP excess in $R_b = \frac{\Gamma(z \to b\bar{b})}{\Gamma(z \to h)}$. Th average value of R_b quoted by the LEP collaborations [1] is $R_b^{expt} \approx .2202 \pm .0020$. This is 2.4σ higher than the SM prediction $R_b^{SM} \approx .2156 \pm .0004$. A somewhat closely related but less prominent dicrepancy is the 1.3σ deficit in the LEP value of $R_c = \frac{\Gamma(z \to c\bar{c})}{\Gamma(z \to h)}$. The experimental value [1] is $rc^{expt} \approx .1583 \pm .0098$ which is to be compared with the SM prediction $R_c^{SM} \approx .1711$. The third discrepancy concerns the difference between the QCD coupling constant $\alpha_s(M_z)$ determined from Z pole measurements and other low energy data [2]. Low energy measurements [2] such as DIS and v decay favor a value of $\alpha_s(M_z)$ close to 0.111. Lattice simulations of the bottomonium system [3] gives $\alpha_s(M_z) \approx .115 \pm .002$ which is consistent with the DIS value. On the other hand high energy measurements at LEP based on $R_l = \frac{\Gamma(Z \to h)}{\Gamma(Z \to l^+ l^-)}$ gives the value [2,4] $\alpha_s(M_z) \approx .128 \pm .005$. A global fit to all the Z line shape data, including R_l, gives $\alpha_s(M_z) \approx .125 \pm .004$. It is therefore puzzling that the LEP values are $\alpha_s(M_z)$ are systematically higher than the low energy values. Among these three deviations, the discrepancy in R_c is least serious first because the LEP deficit is only at the level of 1.3σ and second the charm fragmentation functions are not known that accurately.

Several ETC scenarios that can resolve the $R_b - \alpha_s$ crisis of the SM have been proposed. In the traditional commuting ETC sceario, diagonal ETC exchange tends to increase R_b [5] whereas sideways ETC exchange tends to decrease R_b [6] relative to its SM value. On the other hand in non-commuting ETC scenario [7] the sideways ETC induced vertex correction tends to increase R_b whereas the mixing between the two neutral Z bosons tends to decrease it. The overall size and sign of ETC induced correction δR_b in both these scenarios is therefore model dependent and we cannot make a definite prediction

about R_b. In this article we will consider a commuting ETC scenario whose fermionic representation contains ordinary fermions with V-A weak interaction but TF's with V+A weak interaction and we shall show that it can resolve the $R_b - \alpha_s$ crisis of the SM through sideways ETC exchange only. Unlike standard commuting ETC scenarios there is no need to invoke additional diagonal ETC exchange to get a positive δR_b.

Usually in commuting ETC models LH ordinary fermions ($\psi_L = (t_L, b_L)$) and TF's ($T_L = (U_L, D_L)$) of $I_3 = \pm 1/2$ are placed in identical representations [8] of G_{ETC}. On the other hand to produce the observed isospin breaking in the ordinary fermion mass spectrum (t_R, U_R) are placed in a different representation from (b_R, D_R). In this article we shall deviate somewhat from this requirement and assume that ($\psi_L = (t_L, b_L)$) and $\tilde{T}^c_L = \imath\tau_2 T^c_L = (D^c_L, -U^c_L)$ with $I_3 = \pm 1/2$ are placed in identical representations of G_{ETC}. Here T^c is the charge conjugated TF doublet i.e. $T^C = C\bar{T}^T$. On the other hand (t_R, D^c_R) with $I_3 = 1/2$ will be assumed to be placed in a different representation from (b_R, U^c_R) with $I_3 = -1/2$ to produce the large top-bottom mass splitting. Note that under $SU(2)_w \times U(1)_y$, ψ_L transforms as $(2, 1/6)$ and $\tilde{T}^c_L$ transforms as (2, 0). On the other hand U^c_R and D^c_R trnansform as (1, -1/2) and (1, 1/2) respectively under the same. This implies that TF's have V+A weak interaction. Also since $T^c_L = (U^c_L, D^c_L)$ transforms as 2^* under $SU(2)_w$ it cannot be placed together with ψ_L in the same LH representation of G_{ETC} for a commuting ETC scenario. It is clear from the above that the LH ETC representation does not commute with $U(1)_y$. The corresponding ETC gauge boson $X_{s\mu}$ therefore carries hypercharge. $U(1)_y$ invariance implies that $X_{s\mu}$ can mediate transition between t_R and D^c_R or between b_R and U^c_R but not between t_R and U^c_R or between b_R and D^c_R. Following th usual practice of normalizing charged current interaction describing sideways intraction can be written as

$$L_{ETC} = -\frac{1}{\sqrt{2}}(X_{s\mu} J^\mu_s + h.c.). \tag{1}$$

where $J^\mu_s = g_{E,L}\bar{\psi}_L\gamma^\mu\tilde{T}^c_L - g^U_{E,R}\bar{t}_R\gamma^\mu D^c_R + g^D_{E,R}\bar{b}_R\gamma^\mu U^c_R$. Here color and technicolor indices have been suppressed. The above sideways Lagrangian will give rise to the following masses

for t and b:

$$m_t \approx \frac{g_{E,L} g^U_{E,R} < \bar{D}D >}{2M_s^2} \quad and \quad m_b \approx \frac{g_{E,L} g^D_{E,R} < \bar{U}U >}{2M_s^2}. \tag{2}$$

If the TC sector is isospin symmetric i.e. if $< \bar{U}U > = < \bar{D}D >$, the large heirarchy between m_t and m_b can be produced by ETC interactions provided $g^U_{E,R} \gg g^D_{E,R}$. Since we do not have any realistic ETC model that produces this large heirarchy between m_t and m_b we have simply assumed different sideways couplings of t_R and b_R to the same sideways ETC gauge boson. The product of LH(RH) sideways current with its h.c. gives rise to $Zb_L\bar{b}_L(Zb_R\bar{b}_R)$ vertex correction. However since $\delta\Gamma_b \propto 2[g^b_{L,SM}\delta g^b_L + g^b_{R,SM}\delta g^b_R]$ and $|g^b_{R,SM}| \ll |g^b_{L,SM}|$ we can ignore the effect of δg^b_R on δR_b provided $|\delta g^b_R| \approx |\delta g^b_L|$. The ETC induced correction δg^b_L can be derived from the following 4f sideways interaction

$$L^s_{4f} \approx -({g_{E,L}}^2/2M_s^2)\bar{\psi}_L\gamma^\mu \imath\tau_2 T^c_L \bar{T}^c_L(-\imath\tau_2)\gamma_\mu\psi_L \quad . \tag{3}$$

Fierz transforming both w.r.t Dirac and $SU(2)_w$ indices and using the identities $\bar{T}^c_L\gamma^\mu T^c_L = \bar{T}_R\gamma^\mu T_R$, $\bar{T}^c_L\gamma^\mu\tau^*_i T^c_L = \bar{T}_R\gamma^\mu\tau_i T_R$ we get

$$L^s_{4f} \approx -({g_{E,L}}^2/2M_s^2)[\bar{\psi}_L\gamma_\mu\psi_L\bar{T}_R\gamma^\mu T_R - \bar{\psi}_L\gamma_\mu\tau_i\psi_L\bar{T}_R\gamma^\mu\tau_i T_R]. \tag{4}$$

The sideways ETC induced vertex correction δg^b_L can be obtained from the expression for L^s_{4f} if we replace the TF currents by the corresponding sigma model currents below the TC chiral symmetry breaking scale [9]. We have $\bar{T}_R\gamma_\mu T_R = (\imath F^2_{TC}/2)Tr[\Sigma(D_\mu\Sigma)^+]$ and $\bar{T}_R\gamma_\mu\tau_i T_R = (\imath F^2_{TC}/2)Tr[\Sigma\tau_i(D_\mu\Sigma)^+]$ where $\Sigma = e^{\frac{\imath\tau^i\pi_i}{F_{TC}}}$ and for mirror TF's

$$\begin{aligned} D_\mu\Sigma = \partial_\mu\Sigma - \frac{\imath g}{2\sqrt{2}}\Sigma(W^+_\mu\tau^+ + W^-_\mu\tau^-) + \imath e[Q,\Sigma]A_\mu \\ - \frac{\imath e s_w}{c_w}[Q,\Sigma]Z_\mu - \frac{\imath g}{c_w}\Sigma\frac{\tau_3}{2}Z_\mu. \end{aligned} \tag{5}$$

In the unitary gauge $\Sigma = 1$ and we get $\bar{T}_R\gamma_\mu T_R = 0$ and

$$\bar{T}_R\gamma_\mu\tau_i T_R = -\frac{F^2_{TC}}{2}g(W_{1\mu}\delta_{i1} + W_{2\mu}\delta_{i2}) - \frac{F^2_{TC}}{2}(g/c_w)Z_\mu\delta_{i3}. \tag{6a}$$

The $Z\psi_L\bar{\psi}_L$ vertex correction is therefore given by

$$\delta L_{z\psi_L\bar{\psi}_L} = -({g_{E,L}}^2/8M_s^2)F_{TC}^2(g/c_w)\bar{\psi}_L\gamma_\mu\tau_3\psi_L Z^\mu \qquad (6b)$$

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In SM at tree level the $Zb_L\bar{b}_L$ coupling is given by $L_{Zb_L\bar{b}_L} = -g/c_w g^b_{L,SM} Z_\mu \bar{b}_L\gamma^\mu b_L$ where $g^b_{L,SM} = -(1/2) + (1/3)s_w^2$. Since $\delta g^b_L(sideways) = -\frac{{g_{E,L}}^2 F_{TC}^2}{8M_s^2}$ is of the same sign as $g^b_{L,SM}$, the sideways ETC induced correction in the mirror TF scenario tends to increase Γ_b and hence R_b ($\frac{\delta R_b}{R_b} = (1-R_b)\frac{\delta\Gamma_b}{\Gamma_b}$) relative to its SM value. In standard ETC scenario the sideways exchange produces a negative δR_b and one has to invoke additional diagonal exchange to get an overall positive δR_b to fit ythe LEP data. Whereas in our model the sideways exchange by itself produces a positive δR_b and there is no need to invoke diagonal ETC exchange. The closure of sideways currents however implies the existence of diagonal currents. The diagonal ETC exchange will produce a negative δR_b in our model. The contribution of diagonal exchange can however be made negligible by making the corresponding gauge bosons much heavier than the sideways gauge bosons. For simplicity we shall assume that the TF's are placed in the fundamental representation of $SU(N)_{TC}$. Using naive dimensional analysis [10] and large N_{TC} scaling we can then write $<\bar{D}D> \approx 4\pi F_{TC}^3\sqrt{\frac{3}{N_{TC}}}$. From the xpression of m_t (Eqno. 2) we get $\frac{{g_{E,L}}^2 {F_{TC}}^2}{M_s^2} \approx \frac{m_t}{2\pi F_{TC}}\frac{g_{E,L}}{g^U_{E,R}}\sqrt{\frac{N_{TC}}{3}} \approx .1128\sqrt{\frac{N_{TC}}{3}}\frac{g_{E,L}}{g^U_{E,R}}$ for $m_t \approx 175$ Gev and $F_{TC} \approx 247$ Gev. Further using the relation $\frac{\delta R_b}{R_b(1-R_b)} \approx \frac{\delta\Gamma_b}{\Gamma_b} \approx -4.5758\delta g^b_L$ we find that to produce the 2.4σ LEP excess in R_b, the ratio $\frac{g_{E,L}}{g^U_{E,R}}$ must assume the values .517, .365, .299, and .259 for $N_{TC} =$ 2, 4, 6 and 8 respectively. Hence if the sideways ETC scale is low enough to produce an $m_t \approx 175$ Gev, the values of $\frac{g_{E,L}}{g^U_{E,R}}$ for $N_{TC} \leq 8$ that are required to produce the desired LEP excess in R_b are therefore quite natural and needs no fine tuning.

The contribution of new phyics to $\delta\Gamma_b$ also affects the determination of $\alpha_s(M_z)$ from R_l. Denoting the new physics contribution to Γ_b, $\frac{g_{vl}}{g_{al}}$ and $\alpha_s(M_z)$ by δ^{new}_{vb}, $\delta^{new}(\frac{g_{vl}}{g_{al}})$ and

$\delta\alpha_s^{new}$ respectively we can write to a very good approximation [11]

$$\Gamma_b \approx \Gamma_b^{SM}(1+\delta_{vb}^{new}). \tag{7a}$$

$$\delta R_b \approx \frac{13}{59}[\frac{46}{59}\delta_{vb}^{new} + \frac{24}{767}\delta^{new}(\frac{g_{vl}}{g_{al}}) + 0.1(\delta\alpha_s^{new}(M_z^2)/\pi)]. \tag{7b}$$

and

$$\delta R_l \approx \frac{59}{3}[\frac{13}{59}\delta_{vb}^{new} + \frac{20}{59}\delta^{new}(\frac{g_{vl}}{g_{al}}) + .328\delta\alpha_s^{new}(M_z^2). \tag{7c}$$

where $\delta R_b \approx .0046$ and $\delta R_l \approx .0340$. If we assume that the new physics does not couple directly to leptons we have $\delta^{new}(\frac{g_{vl}}{g_{al}}) = 0$. From eqns. (7b) and (7c) we then get $\delta_{vb}^{new} \approx .0277$ and $\delta\alpha_s^{new}(M_z^2) \approx -.0133$. From the experimental value of R_l on can determine $\alpha_s(M_z^2)$ in the context of the SM in a relatively clean way. The value obtained in this way is $\alpha_s^{SM}(M_z^2) \approx .128 \pm .005$. Incorporating the correction due to new physics we get $\alpha_s^{new}(M_z^2) \approx .115$ which is in excellent agreement with the Lattice determination of $\alpha_s(M_z^2) \approx .115 \pm .002$ from the υ system. It also agrres with the value of $\alpha_s(M_z^2)$ determined from DIS ($.113 \pm .005$) within its uncertainties.

The low ETC scale that gives rise to the large mass for the top quark can also produce observable weak isospin breaking effects [12]. In our model there are two ETC induced 4f operators that lead to weak isospin breaking.

$$O_1 = \frac{{g_{E,L}}^2}{4M_s^2}[\bar{\psi}_L\gamma_\mu\tau_i\psi_L\bar{T}_R\gamma^\mu\tau_i T_R - \bar{\psi}_L\gamma_\mu\psi_L\bar{T}_R\gamma^\mu T_R]. \tag{8a}$$

and

$$O_2 = -\frac{(g_{E,R}^U)^2}{2M_s^2}[\bar{D}_L\gamma_\mu D_L\bar{t}_R\gamma^\mu t_R]. \tag{8b}$$

In the above we have assumed that the product of two I=1 TF currents that usually lead to the most dangerous weak isospin violation [12] in TC models is subdominant. The reason being such potentially dangerous operators arise from diagonal ETC exchange but the mass of such diagonal gauge bosons have been assumed to be much greater than that of sideways gauge bosons. The terms in O_1 and O_2 that contain either an isosinglet TF

current or an isosinglet ordinary fermion current does not contribute to $\Pi^{11}(q^2)$ or $\Pi^{33}(q^2)$ and therefore they can be dropped. Let $\tilde{\Pi}^{i,j}_{V,A}(q^2)$ and $\bar{\Pi}^{i,j}_{V,A}(q^2)$ denote the gauge boson self energy correction due to TF's and ordinary fermions respectively. It can then be shown that the contribution of O_1 to $\Pi^{11}(0)-\Pi^{33}(0)$ is given by $[\Pi^{11}(0)-\Pi^{33}(0)]_{O_1} = -\frac{3g^2_{E,L}F^2_{TC}m^2_t}{256\pi^2 M^2_s}$. To find the contribution of O_2 to $\Pi^{11}(0)-\Pi^{33}(0)$ we shall make use of the fact that $\tilde{\Pi}^{33}_V(0) = \bar{\Pi}^{33}_V(0) = 0$ due to exact conservation of neutral vector current. We then have $[\Pi^{11}(0)-\Pi^{33}(0)]_{O_2} = \frac{3(g^U_{E,R})^2F^2_{TC}m^2_t}{256\pi^2 M^2_s}\ln\frac{\Lambda^2_s}{m^2_t}$ where $\Lambda^2_s \approx \frac{M^2_s}{g^2_{E,L}}$. Hence $\delta\rho_s \approx \frac{3m^2_t}{64\pi^2 M^2_s}[(g^U_{E,R})^2\ln\frac{\Lambda^2_s}{m^2_t} - g^2_{E,L}] \approx .0031$. From a global fit of the LEP data one obtains the bound [1] $\delta\rho^{expt}_{new} \leq .4\%$. The sideways ETC induced correction $\delta\rho_s$ is therefore barely consistent with the global fits to the LEP data.

Although the mirror TF scenario presented here provides a better fit to the LEP data w.r.t δR_b and α_s, it does not produce the necessary shift in R_c to account for the observed LEP deficit. If the parametrs of the ETC model are so chosen as to produce the observed LEP excess in R_b, the corresponding shift in R_c is given by $\delta R_c \approx -R_c\frac{\delta R_b}{1-R_b} \approx -.0011$, which is only 8.59% of the desired shift of -.0128. A toy grand unified model containing four ordinary fermion families with V-A interaction and four TF families with V+A interaction can be constructed based under the group $O'(14) = SO(14)\times K'$ where $K'(64_L) = 1$ and $K'(\bar{6}4_R) = -1$. The irreducible spinorial representation 128 of O'(14) decomposes under $SO(10)\times SU(2)_H \times SU(2)_{TC}$ as follows: $128 = (16_L, 2, 1) \oplus (\bar{1}6_R, 2, 1) \oplus (\bar{1}6_L, 1, 2) \oplus (16_R, 1, 2)$.

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